On The Structure of $A = 3$ Nuclei

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The hole in the charge distribution of ^{3}He is a major problem in $A = 3$ nuclei. The canonical wavefucntion of $A = 3$ nuclei which does well for electromagnetic properties of $A = 3$ nuclei fails to produce the hole in $A = 3$ nuclei. The hole is normally assumed to arise from explicit quark degree of freedom. Very often quark degrees of freedom are imposed to propose a different short range part of the wavefunction for $A = 3$ to explain the hole in ^{3}He. So an hybrid model with nucleonic degree of freedom in outer part and quark degrees of freedom in the inner part of the nucleus have been invoked to understand the above problem. Here we present a different picture with a new wavefunction working at short range within nucleonic degrees of freedom itself. So the above problem is explained here based entirely on the nucleonic degree of freedom only.

Clusters are playing important role in current nuclear physics. Of these α-clusters are known to play a prominent role. These are known to be spontaneously emitted from heavy nuclei. Heavier clusters radioactivity was predicted by Sandulescu, Poenaru and Greiner in 1980 [1]. This was subsequently confirmed a couple of years later and several heavy clusters like ^{14}C, ^{24}Ne, ^{28}Mg, ^{32}Si etc. have been observed and well studied [2].

Other clustering in particular $A = 3$ clustering is also possible and this was recently studied in detail by us [3]. Strong experimental evidences of clustering of $A = 3$ nuclei- ^{3}He(Helion) and ^{3}H(Triton) were pointed out and a consistent theoretical understanding was attempted therein. One would have thought that a nuclei as simple as $A = 3$ should have been well understood by now. But in keeping with complexity in the structure of another three body system, that of baryons made up of three quarks, things are not all that easy.

To understand the clustering of $A = 3$ nuclei we have to see what aspects are the ones which contribute to its clustering property. Hence a better and consistent understanding of the structure of $A = 3$ is required.

To understand as to what makes $A = 3$ so special let us study the properties of the ground state magnetic moment of ^{3}He and ^{3}H, and the ^{3}He charge density.

The degrees of freedom relevant for ^{3}He and ^{3}H are spin, isospin and orbital space. The total wavefunction should be antisymmetric. For three nucleons the spin wave function in standard notation are of mixed symmetry χ_ρ and χ_λ where ρ and λ correspond to mixed symmetric state with the first two spins in antisymmetric and symmetric states respectively. The corresponding mixed symmetric state for the three nucleons are ϕ_ρ and ϕ_λ. If we ensure full antisymmetry in spin-isospin space then

$$\Psi_{\mathrm{A}} = \frac{1}{\sqrt{2}} \left(\chi_\lambda \phi_\rho - \chi_\rho \phi_\lambda \right) \tag{1}$$

then the orbital part of the wavefunction for $A = 3$ should be completely symmetric which we take as Fadeev wavefunction [4]

$$\Psi_{\mathrm{A=3}} = \frac{\alpha^3}{\pi^{3/2}} e^{-\alpha^2 \left(\rho^2 + \chi^2 \right)/2} \tag{2}$$

where

$$\begin{aligned}
\vec{\rho} &= \left(\vec{r}_1 - \vec{r}_2 \right) / \sqrt{2} \\
\vec{\lambda} &= \left(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3 \right) / \sqrt{6}
\end{aligned} \tag{3}$$

So the three nucleons are in the ground state are in orbital symmetric S-state. Pauli Principle require that in $A = 3$, the two like nucleons would be in spin zero state. Then the magnetic dipole moment of ^{3}H would be entirely due to the odd proton and also would arise from the odd neutron in ^{3}He.

Therefore this picture predicts

$$\begin{aligned}
\mu \left(^3\mathrm{H} \right) &= \mu \left(p \right) = 2.79 \text{ nm} \\
\mu \left(^3\mathrm{He} \right) &= \mu \left(n \right) = -1.91 \text{ nm}
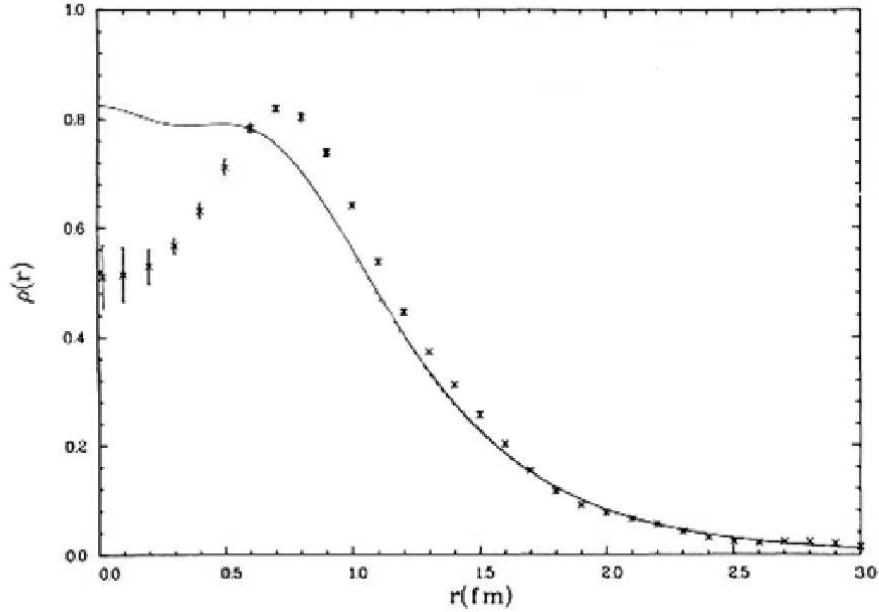\end{aligned} \tag{4}$$

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FIG. 1: Point nucleon ^{3}He theoretical charge density and the corresponding experimental density indicated as data points[4].

This is in good agreement with the experimental value of 2.97 nm and -2.12 nm, respectively. Thus clearly a symmetric space wavefunction in S-state does reasonably well for the magnetic moment of $A = 3$ nuclei.

What will such a wavefunction predict for the charge distribution for ^{3}He for example. Using the symmetric space wavefunction (2) the proton nuclear charge distribution are given in Fig. 1. This is plotted against experimental results [4]. Such a wavefunction does reasonably well for $r \geq 1$ but fails miserably for small r. Experimentally there is a 'hole' in the center of ^{3}He charge distribution. So, the symmetric space part of the wavefunction which does do well for the magnetic dipole moment of $A = 3$ nuclei is failing badly at small distances of the charge distribution. How to accommodate this?

One technique is to split the wavefunction in two parts. For some r_0 for $r > r_0$ the wavefunction is assumed to look like that of (1) and (2) while for $r < r_0$ it is assumed that quark degree of freedom may play a role [5, 6]. At small relative distances the nucleons are likely to overlap strongly and as such probability of existence of 6 quark [5] clusters or 9 quark [6] clusters towards the centre of ^{3}He would be non-negligible and thus the hole may be explained as a consequence of these quark degrees of freedom in $A = 3$ nuclei (note ^{3}H also has a similar distribution with a hole at the centre [6]).

In this way the short range part of the wavefunction is arising due to the explicit quark degrees of freedom. In this paper we ask the question- Is it possible to understand this hole in $A = 3$ nuclei within only the nucleonic degrees of freedom without invoking the quark degrees of freedom. Below we show that indeed this is possible!

We accept that there are two parts of the $A = 3$ wavefunction. For some r_0 , for $r > r_0$ let the wavefunction be described in the canonical way through equations (1) and (2). This is good as it explains the $A = 3$ magnetic dipole moment satisfactorily.

For nucleons in $A = 3$ the degree of freedom are spin-isospin and orbital part. In equation (1) the spin-isospin part was antisymmetric and thus the space part was symmetric (2).

Note that it is possible to have an entirely symmetric spin-isospin wavefunction for the $A = 3$ system

$$\Psi_{\mathrm{S}} = \frac{1}{\sqrt{2}} \left(\chi_\lambda \phi_\lambda + \chi_\rho \phi_\rho \right) \tag{5}$$

If this holds then what we require is antisymmetric space wavefunction. So the question is- is it possible to have an antisymmetric part of space wavefunction for the ground state of $A = 3$ nuclei?

We take the cue from quark model of hadrons. As is well known [7] nucleons are made up of three constituent quarks. It was found that in space-isospin-orbital space a symmetric wavefunction worked well. Normally one would have a symmetric $L = 0$ state for the ground state and in which case it was a puzzle as to how the three spin-1/2 quarks requires symmetric spin-isospin-orbital wavefunction. One may fix for this by involving an additional color degree of freedom to carry the antisymmetry. With hindsight, this was the correct way to solve the problem. But

in the early days of quark model, without color degree of freedom, one possibility was to seek for an $L = 0$ orbital antisymmetric wavefunction. And indeed it is possible to construct $L = 0$ wavefunction for baryons which is totally antisymmetric [7]. Though in keeping with Pauli principle this was possible, a difficulty was pointed out by Mitra and Majumdar [8]. What they showed was that with such an antisymmetric orbital wavefunction for 3 quarks in $L = 0$ state there would be zeros in the form factors of proton, and such a situation does not arise experimentally. So this ruled out the possibility of absorbing antisymmetry of 3 quarks in the orbital space in quark model [7–9]. And as we know in quark model- this problem was solved through the correct invocation of a new three color degrees of freedom.

Here for us the $L = 0$ antisymmetric wavefunction of three fermions is significant. If for $A = 3$ the spin-isospin wavefunction is symmetric as in Eq.(5) then the orbital part should be antisymmetric here. What failed for 3 quarks in baryons works wonderfully for 3 nucleons in $A = 3$ nuclei. We see this below.

Let the completely antisymmetric wavefunction in orbital space be given as $f(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ [7][p.31] (the exact form of f is not important for our discussion here). Then the charge density is

$$\rho(\vec{r}) = \int d^3\vec{r}_2 \ |f(\vec{r}, \vec{r}_2, -(\vec{r} + \vec{r}_2))|^2 \tag{6}$$

We choose the coordinate of the three nucleons in the center of mass system in which $\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0$. As f is antisymmetric the integrand in Eq.(6) vanishes and thus the charge density at the origin is zero.

Thus the existence of antisymmetric orbital wavefunction for $A = 3$ demands that the charge density vanishes at $r = 0$ for these nuclei. So the wavefunction predicts a hole at the centre of $A = 3$ nuclei exactly as found for ^{3}He (Fig.1). One may expect that the finite size of nucleons would smear out the hole somewhat but should still leave a prominent depression in the density near the center which would be seen as a hole (see Fig.1).

Thus we suggest that for $r < r_0$ (where $r_0 \sim 1$fm) the wavefunction is $\Psi_{\text{Symmetric}}$(spin-isospin) $\times$ $\Psi_{\text{Antisymmetric}}$(orbital) and thus the hole is naturally predicted by such a wavefunction.

Note the $\langle \Psi_A | \Psi_S \rangle = 0$ (from Esq. (1) and (5)) and thus the two parts of the wavefunction, one for $r > r_0$ (for some suitable values say $r_0 \sim 1$fm for ^{3}He) and the other $r < r_0$ with Eq.(5) would work well for both the magnetic moment etc. and for reproducing charge/matter densities of $A = 3$ nuclei, and thus is entirely within the nucleonic degree of freedom only.

Hence both antisymmetric and symmetric orbital parts play a complementary role in predicting the magnetic moment as well as the hole in $A = 3$nuclei. It is like two different phases (with orbital part antisymmetric in one and completely different that is symmetric, in the other one) coexisting for the same nucleus $A = 3$.

Note that light nuclei $A = 1$ and $A = 2$ are all pure 'surface' nuclei. A genuine nucleus should have a distinct 'interior' plus a clear 'surface' outside. $A = 3$ is hence, the first genuine nucleus. It has managed to build an "interior" while retaining the "surface". And the new structure suggested here plays a fundamental role in creating a first clear cut "interior" (which appears as a hole) while still retaining the "surface" outside.

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[1] A, Sandulescu, D. N. Poenaru and W. Greiner, Sov. J. Part. Nucl. **11C**, 528 (1980).
[2] D. N. Poenaru, R. A. Gherghescu and W. Greiner, Phys. Rev. **C83**, 014601 (2011).
[3] S. A. Abbas and Shakeb Ahmad, Int. J. Mod. Phys. **E20**, 2101 (2011).
[4] J. L. Friar, B. F. Gibson, E. L. Tomusiak and G. L. Payne, Phys. Rev. **C24**, 665 (1981).
[5] P. Hoodbhoy and L. S. Kisslinger, Phys. Lett. **B146**, 163 (1984).
[6] A. Abbas, Phys. Lett. **167**, 150 (1986).
[7] J. J. J. Kokkedee, "The Quark Model", Benjamin, New York (1979).
[8] A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).
[9] R. E. Kreps and J. J. de Swart, Phys. Rev. **162**, 1729 (1967).